



Calculations Policy

**Penkridge Middle School
Princefield First School
Marshbrook First School
St. Michael's C.E.(A) First School
St. Leonard's C.E. (VC) First School
Brewood Middle C.E. (C) School
St Mary and St Chad C.E. (VC) First School
St John's C.E. (VC) First School, Bishops Wood
St Paul's CE (VC) First School, Coven
St Mary's C of E (C) First School
Wolgarston High School**

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* Mental and Written methods



Introduction

Progression for Addition, Subtraction, Multiplication and Division.

These notes and illustrations show individual methods for stages of progression that build to a compact, efficient method (standard method) for all four operations.

The aim of this policy is:

- all children to use effective mental strategies when appropriate
- they choose an appropriate written method which they can use accurately and with confidence.

Time must be taken to build up to the most efficient method to ensure complete understanding at each stage.

As a guide, when children arrive at Penkridge/Brewood Middle School they should be:

Addition - starting stage 6

Subtraction - starting stage 5

Multiplication - starting stage 9

Division - starting stage 6

As a guide, by the end of year 6 children should be at the following stages:

Addition - stage 8

Subtraction - stage 8

Multiplication - stage 12

Division - stage 8

Children should not move onto the next stage if:

- 1) they are not ready i.e. lack of understanding of place value.
- 2) they are not confident with previous stages

If the children can complete these stages they will have a range of effective methods for mental and written calculations.

Maths Language

Children should experience the various maths language used within the four operations for mental and written questions.

Symbol	Words Used
+	<u>Addition</u> , Add, Sum, Plus, Increase, Total, more, together, and
-	<u>Subtraction</u> , Subtract, Minus, Less, Difference, Decrease, Take Away, Deduct, fewer, take from, reduce
×	<u>Multiplication</u> , Multiply, Product, Multiplied By, Times, Lots of, groups of, times tables
÷	<u>Division</u> , Divide, divided by, share, share equally, divisible by, divide into, group, Quotient (is the result of division)

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of number bonds

$$6 + 4 = 10$$

$$\square + 3 = 10$$

$$25 + 75 = 100$$

$$19 + \square = 20$$

Children should know number bonds for all multiples of 10

Use doubles and near doubles of small numbers

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

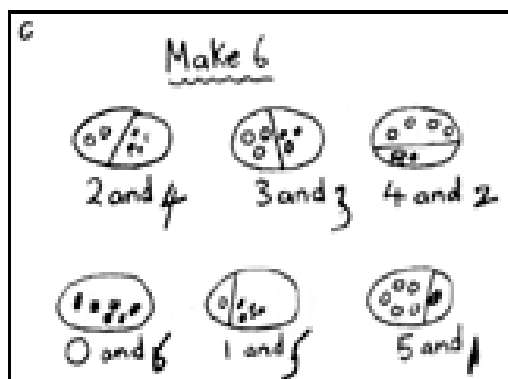
$$55 - 36 = 19$$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

THE FOLLOWING ARE METHODS THAT WE EXPECT THE MAJORITY OF CHILDREN TO BE ABLE TO ACHIEVE.

Stage 1

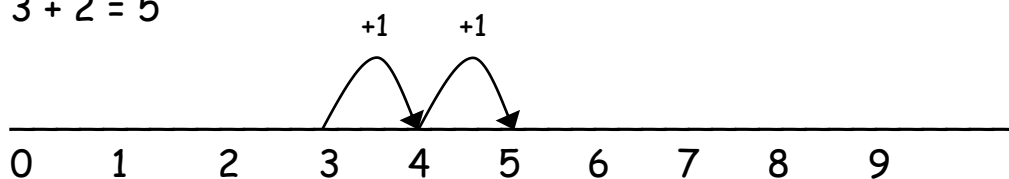
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, rhymes, songs etc.



Stage 2

They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the numberline.

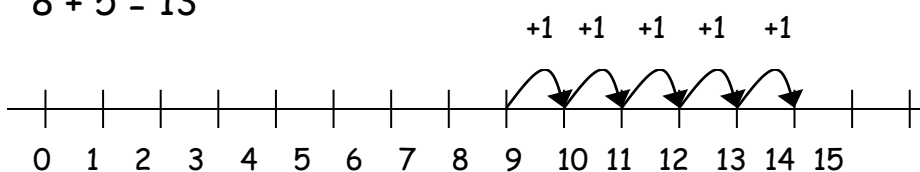
$$3 + 2 = 5$$



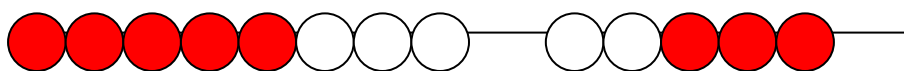
This stage could also be supported with a number grid/100 square

Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

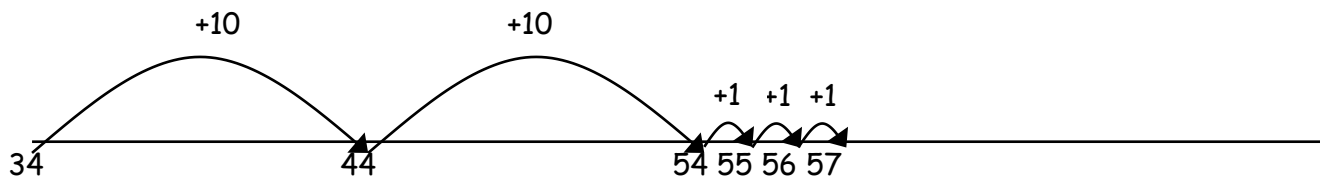


Stage 3

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

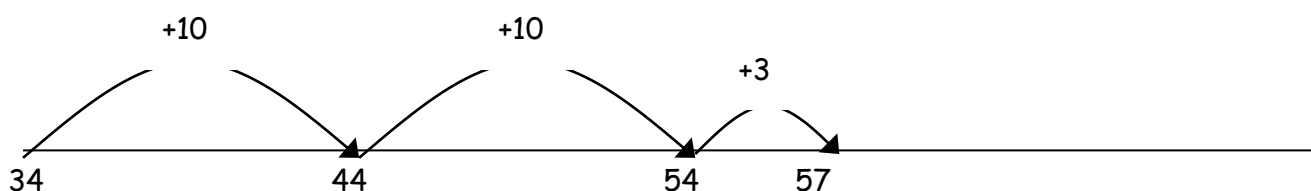
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



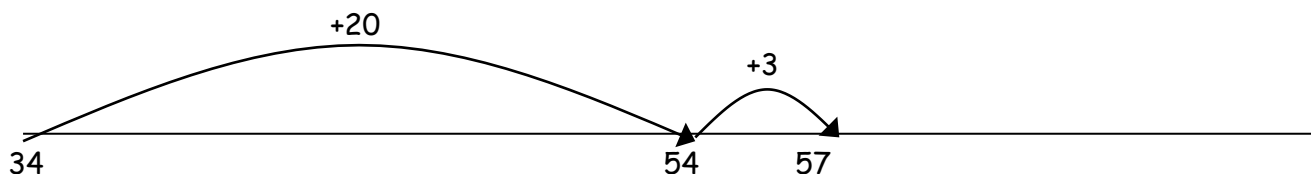
- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



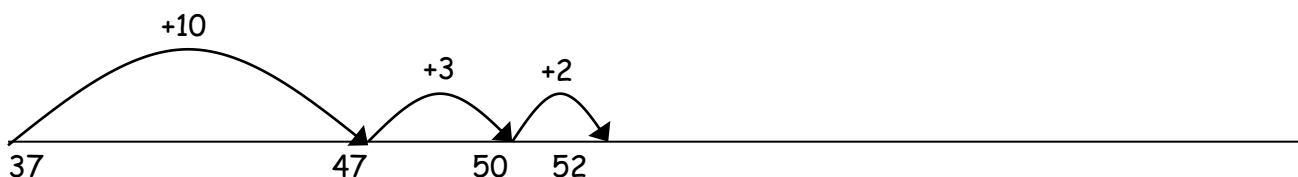
- ✓ Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



- ✓ Bridging through ten can help children become more efficient.

$$37 + 15 = 52$$



Stage 4

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

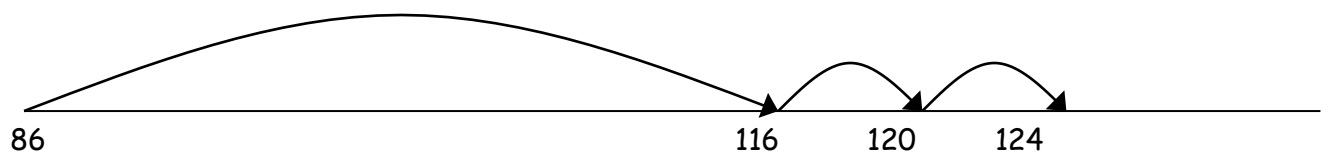
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$

+30

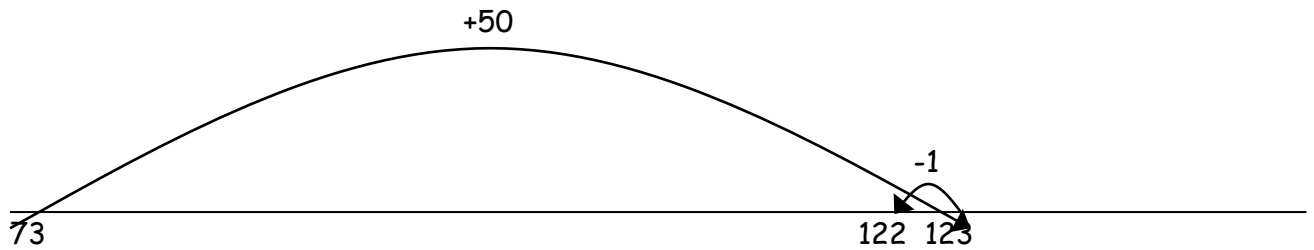
+4

+4



- ✓ Compensation by rounding to the nearest 10 and take away the number they had added when rounding.

$$49 + 73 = 122$$



Stage 5

Formal Methods - Addition uses partitioning.

Option 1 - Adding most significant digits first then, moving to adding least significant digits.

$$\begin{array}{r}
 67 \\
 + 24 \\
 \hline
 80 \text{ (60 + 20)} \\
 \underline{11} \text{ (7 + 4)} \\
 \hline
 91
 \end{array}$$

$$\begin{array}{r}
 267 \\
 + 85 \\
 \hline
 200 \\
 140 \text{ (60 + 80)} \\
 \underline{12} \text{ (7 + 5)} \\
 \hline
 352
 \end{array}$$

Moving to adding the least significant digits first in preparation for 'carrying'.

$$\begin{array}{r}
 67 \\
 + 24 \\
 \hline
 11 \text{ (7 + 4)} \\
 \underline{80} \text{ (60 + 20)} \\
 \hline
 91
 \end{array}$$

$$\begin{array}{r}
 267 \\
 + 85 \\
 \hline
 12 \text{ (7 + 5)} \\
 140 \text{ (60 + 80)} \\
 \underline{200} \\
 \hline
 352
 \end{array}$$

Option 2 - Adding the least significant digits first

$$\begin{array}{r}
 67 \\
 + 24 \\
 \hline
 11 \text{ (7 + 4)} \\
 \underline{80} \text{ (60 + 20)} \\
 \hline
 91
 \end{array}$$

$$\begin{array}{r}
 267 \\
 + 85 \\
 \hline
 12 \text{ (7 + 5)} \\
 140 \text{ (60 + 80)} \\
 \underline{200} \\
 \hline
 352
 \end{array}$$

Option 1 can be useful for securing place value but if appropriate children can start at option 2

Stage 6

Addition using the standard method (short method)

From this, children will begin to carry below the line.

$$\begin{array}{r}
 625 \\
 + 48 \\
 \hline
 673 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 783 \\
 + 42 \\
 \hline
 825 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 367 \\
 + 85 \\
 \hline
 452 \\
 \hline
 11
 \end{array}$$

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*

- ✓ know that the decimal points should line up under each other, particularly when adding mixed amounts, e.g. £3.59 + 78p.

Stage 7

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ 11 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 111 \end{array}$$

Using similar methods, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm.

Stage 8

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ 111 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ 111 \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ \hline + 4681 \end{array}$$

11944

121

Using similar methods, children will

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimal fractions with up to four digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding mixed amounts, e.g. 401.2 + 26.85 + 0.71.

When using mixed numbers children need to understand that place holders do not change the value of the number.

$$\begin{array}{r} 401.20 \\ 26.85 \\ 0.71 \\ \hline 428.76 \\ \hline 1 \end{array}$$

The children need to be aware that it does not change the value of the number, just supports with lining up the place value.

+ - + - + - + - + - + - +

By the end of all the stages, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 3) they are not ready i.e. lack of understanding of place value.
- 4) they are not confident with previous stages

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

THE FOLLOWING ARE METHODS THAT WE EXPECT THE MAJORITY OF CHILDREN TO BE ABLE TO ACHIEVE.

Stage 1

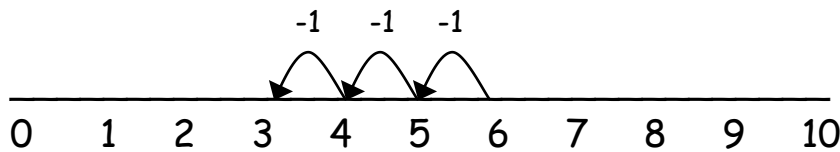
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.



Stage 2

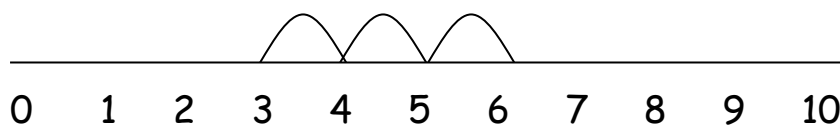
They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

$$6 - 3 = 3$$



This stage could also be support with a number grid/100 square

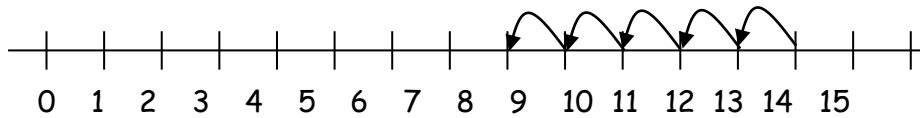
The numberline should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

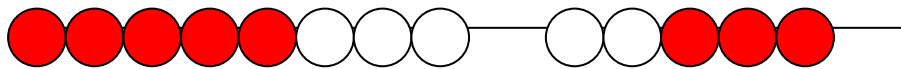
$$13 - 5 = 8$$

-1 -1 -1 -1 -1



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



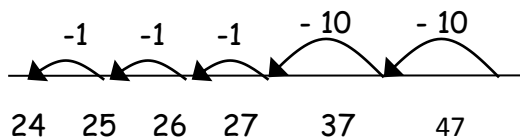
Stage 3

Children will begin to use empty number lines to support calculations.

Counting back

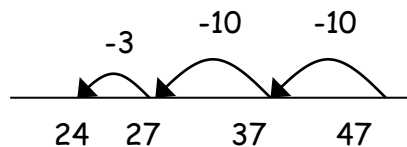
- ✓ First counting back in tens and ones.

$$47 - 23 = 24$$



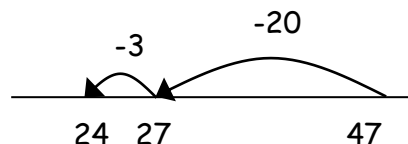
- ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



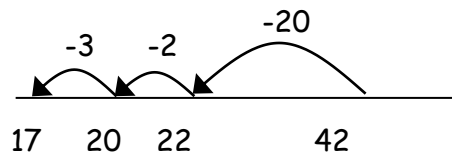
- ✓ Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓
- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



Stage 4

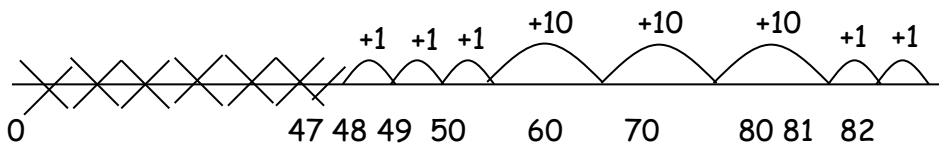
Counting on - difference between the numbers

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

$$82 - 47$$



Help children to become more efficient with counting on by:

- ✓ Subtracting the units in one jump;
- ✓ Subtracting the tens in one jump and the units in one jump;
- ✓ Bridging through ten.

Children will continue to use empty number lines with increasingly large numbers.

Stage 5

Partitioning and decomposition

This process can be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NOTE When solving the calculation $89 - 57$, children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 + 9 \\ \underline{50 + 7} \\ 30 + 2 = 32 \end{array}$$

Initially, the children will be taught using examples that do not need the children to exchange.

From this the children will begin to exchange.

$$\begin{array}{r} 71 \\ - 46 \\ \hline \end{array}$$

$$\text{Step 1} \quad \begin{array}{r} 70 + 1 \\ - 40 + 6 \\ \hline \end{array}$$

$$\text{Step 2} \quad \begin{array}{r} 60 + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

The calculation should be read as e.g. take 6 from 1.

This would be recorded by the children as

$$\begin{array}{r} 60 \\ \cancel{70} + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

Stage 6

Partitioning and decomposition H T U's

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array} =$$

$$\text{Step 1} \quad \begin{array}{r} 700 + 50 + 4 \\ - \quad \quad 80 + 6 \\ \hline \end{array}$$

$$\text{Step 2} \quad \begin{array}{r} 700 + 40 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from } T \text{ to } U)$$

$$\text{Step 3} \quad \begin{array}{r} 600 + 140 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from } H \text{ to } T)$$

$$600 + 60 + 8 = 668$$

This would be recorded by the children as

$$\begin{array}{r}
 \overset{600}{\cancel{700}} + \overset{140}{\cancel{50}} + 14 \\
 - \quad \quad \quad \underline{80 + 6} \\
 \hline
 600 + 60 + 8 = 668
 \end{array}$$

Stage 7

Decomposition - standard method

$$\begin{array}{r}
 6141 \\
 \cancel{754} \\
 - 286 \\
 \hline
 468
 \end{array}$$

$$\begin{array}{r}
 5131 \\
 \cancel{6467} \\
 - 2684 \\
 \hline
 3783
 \end{array}$$

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *to know that decimal points should line up under each other.*
- ✓ *be able to subtract with exchanges*

Stage 8

Partitioning and Decomposition - decimal numbers

- ✓ *using this method, children should also begin to find the difference between decimal numbers*
- ✓ *know that decimal points should line up under each other.*

For example:

$$\begin{array}{r}
 89.5 \\
 - 43.8 \\
 \hline
 \end{array}
 = \begin{array}{r}
 80 + 9 + 0.5 \\
 - 40 + 3 + 0.8 \\
 \hline
 \end{array}$$

leading to

1

$$\begin{array}{r}
 = 80 + 8 + 0.15 \quad (\text{adjust from } T \text{ to } U) \\
 80.85 \\
 - 40 + 3 + 0.8 \\
 \hline
 40.38 \\
 40 + 5 + 0.7 \\
 \hline
 = 40.57
 \end{array}$$

=

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other.*

Stage 9

NB If children have reached the concise stage they will then continue with this method. They will not go back to using the expanded methods.

Decomposition - standard method (decimals)

$$\begin{array}{r} \\ 646.7 \\ - 268.4 \\ \hline 378.3 \end{array}$$

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other.*
- ✓ *Using this method, children should also begin to find the difference between two three-digit sums of money, with or without exchanges from the pence to the pounds*

For example:

$$\begin{array}{r} \pounds 8.95 \\ -\pounds 4.38 \\ \hline \pounds 4.57 \end{array}$$

+ - + - + - + - + - +

By the end of all the stages, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 5) they are not ready i.e. lack of understanding of place value.
- 6) they are not confident with previous stages

Figure 1 PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

Stage 1
2 times table
5 times table
10 times table

Stage 2
2 times table
3 times table
4 times table
5 times table
6 times table
10 times table

Stage 3

Derive and recall quickly all multiplication facts up to 10×10

Stage 4

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$$\begin{aligned} 13 \times 11 &= (13 \times 10) + (13 \times 1) \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

Multiplying by 10 or 100 or 1000

Knowing that the effect of multiplying by 10 is a move in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a move in the digits two places to the left.

Knowing that the effect of multiplying by 1000 is a move in the digits three places to the left.

Partitioning

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 102 \end{aligned}$$

Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

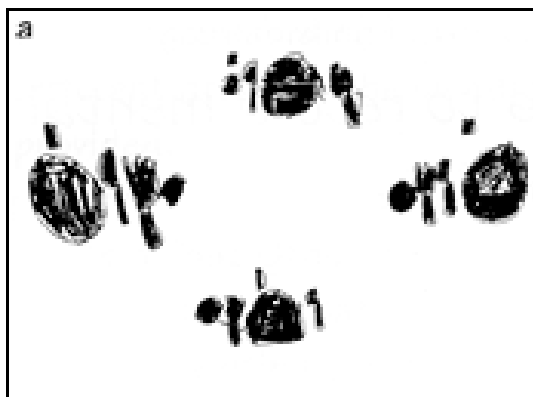
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Children need to be reminded that the decimal point does not move, it's the digits that move into different place values.

THE FOLLOWING ARE METHODS THAT WE EXPECT THE MAJORITY OF CHILDREN TO BE ABLE TO ACHIEVE.

Stage 1

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



Children need visual examples to understand multiplication. Children can group items in a picture or solid items in dishes. When confident with grouping items together, they can then move on to stage 2 by using numbers as well as pictures to link the two together the two stages together

Stage 2

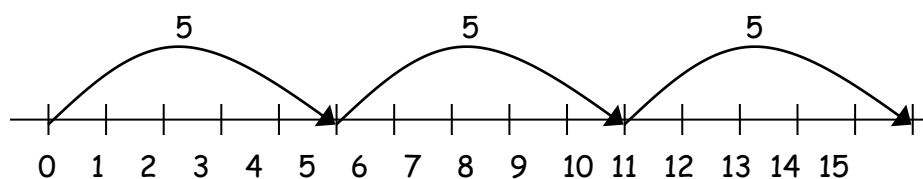
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

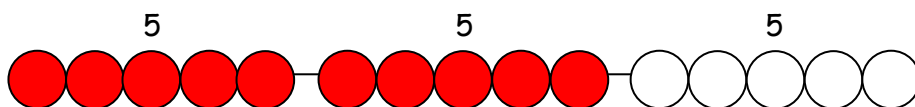
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



and on a bead bar:

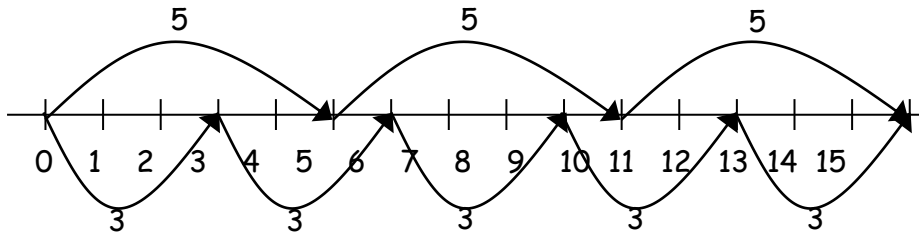
$$5 \times 3 = 5 + 5 + 5$$



Stage 3

✓ **Commutativity**

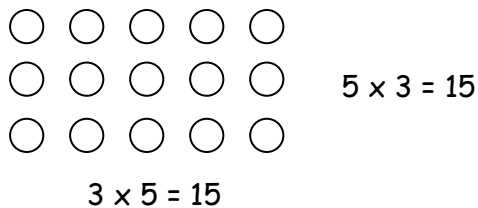
Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



Stage 4

✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

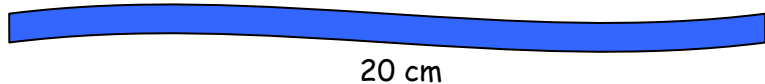
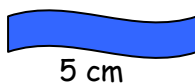


Stage 5

Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



Stage 6

- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times 4 = 32$$

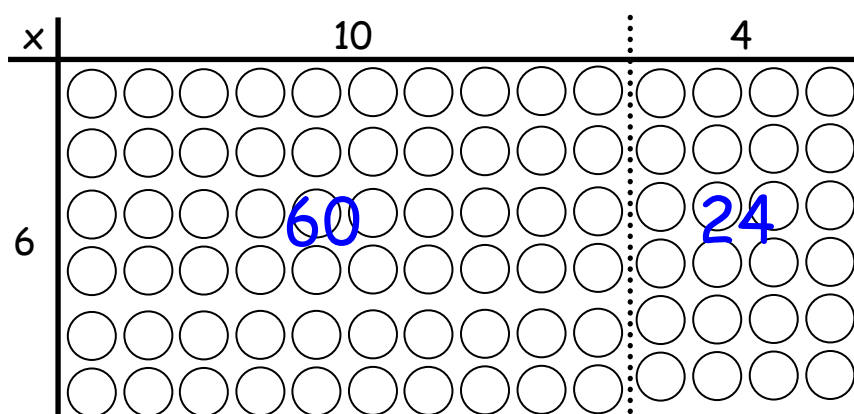
Stage 7

- ✓ Partitioning

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

Stage 8

Children will continue to use arrays where appropriate leading into the grid method of multiplication.



$$(6 \times 10) + (6 \times 4)$$

$$60 + 24$$

$$84$$

This method can be used to visually demonstrate area of shape.

Stage 9

Grid method

TU × U

$$23 \times 8$$

Children will approximate first
 23×8 is approximately $25 \times 8 = 200$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad \boxed{24} \\ \hline 160 \\ + \quad 24 \\ \hline \underline{184} \end{array}$$

Grid method

HTU × U

$$346 \times 9$$

Children will approximate first
 346×9 is approximately $350 \times 10 = 3500$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \\ \hline 2700 \\ + \quad 360 \\ + \quad 54 \\ \hline \underline{3114} \\ \small 11 \end{array}$$

TU × TU

$$72 \times 38$$

Children will approximate first
 72×38 is approximately $70 \times 40 = 2800$

| | | |
|----|------|----|
| x | 70 | 2 |
| 30 | 2100 | 60 |
| 8 | 560 | 16 |

$$\begin{array}{r}
 2100 \\
 + 560 \\
 + 60 \\
 + \underline{16} \\
 \hline
 2736 \\
 1
 \end{array}$$

Grid Method

HTU x TU

$$372 \times 24$$

Children will approximate first
 372×24 is approximately $400 \times 25 = 10000$

| | | | |
|----|------|------|----|
| x | 300 | 70 | 2 |
| 20 | 6000 | 1400 | 40 |
| 4 | 1200 | 280 | 8 |

$$\begin{array}{r}
 6000 \\
 + 1400 \\
 + 1200 \\
 + 280 \\
 + 40 \\
 + \underline{8} \\
 \hline
 8928 \\
 1
 \end{array}$$

Stage 10

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

e.g. 4.9×3

Children will approximate first
 4.9×3 is approximately $5 \times 3 = 15$

$$\begin{array}{r}
 \times \quad 4 \quad 0.9 \\
 3 \quad \boxed{12 \quad 2.7} \\
 \hline
 12 \\
 + \quad 2.7 \\
 \hline
 14.7
 \end{array}$$

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

$$4.92 \times 3$$

Children will approximate first
 4.92×3 is approximately $5 \times 3 = 15$

$$\begin{array}{r}
 \times \quad 4 \quad 0.9 \quad 0.02 \\
 3 \quad \boxed{12 \quad 2.7 \quad 0.06} \\
 \hline
 12 \\
 + \quad 0.7 \\
 + \quad 0.06 \\
 \hline
 12.76
 \end{array}$$

Stage 11

Short Multiplication

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

$$\begin{array}{r}
 38 \\
 \times \quad 7 \\
 \hline
 266 \\
 5
 \end{array}$$

Children need to understand that the number they are multiplying by, multiplies into each number starting with the lowest place value.

Stage 12

Long Multiplication

56×27 is approximately $60 \times 30 = 1800$.

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 1120 \\ \underline{392} \\ 1512 \\ 1 \end{array} \quad \begin{array}{l} 56 \times 20 \\ 56 \times 7 \end{array}$$

- Children need to remember that the number they are multiplying by, multiplies into each number starting with the lowest place value.
- Children start with the tens then move onto the units.
- Children can place a zero in the units place of the tens row but they must be aware that the row is multiply of 10

Children who are already secure with multiplication for TU \times U and TU \times TU should have little difficulty in using the same method for HTU \times TU.

286×29 is approximately $300 \times 30 = 9000$.

$$\begin{array}{r} 286 \\ \times 29 \\ \hline 5720 \\ \underline{2574} \\ 8294 \\ 1 \end{array} \quad \begin{array}{l} 286 \times 20 \\ 286 \times 9 \end{array}$$

This method can be used for decimals by making the numbers being used into whole numbers first by multiplying by 10 and then dividing the answer the answer by 10.

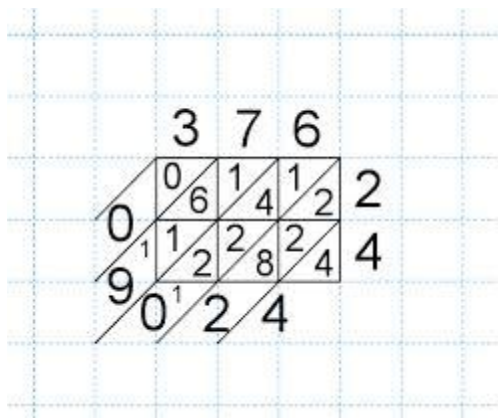
i.e

$$8.6 \times 6 = \quad 8.6 \times 10 = 86 \times 6 = 516 \div 10 = 51.6$$

$$\begin{array}{r} 86 \\ \times 6 \\ \hline 516 \\ 3 \end{array} \quad 86 \times 6$$

Stage 12

Napier's Bones - Chinese Multiplication



If you are multiplying 3 digits by 2 digits you need a 3 by 2 grid which is then split with diagonals lines. Each square represents the numbers on the outside multiplied together. When all the numbers have been multiplied you then add together the numbers in the diagonal lines to give you the answer.

+ - + - + - + - + - + - +

By the end of all the stages, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 7) they are not ready i.e. lack of understanding of place value.
- 8) they are not confident with previous stages

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS

(Ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught every day, either as part of the mental oral starter or other times as appropriate within the day.

2 times table

5 times table

10 times table

Then

3 times table

4 times table

6 times table

Derive and recall quickly division facts for all tables up to 10×10

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Dividing by 10 or 100 or 1000

Knowing that the effect of dividing by 10 is a move in the digits one place to the right.

Knowing that the effect of dividing by 100 is a move in the digits two places to the right.

Knowing that the effect of dividing by 1000 is a move in the digits two places to the right

Use of factors

$$\begin{array}{l} 378 \div 21 \quad 378 \div 3 = 126 \quad 378 \div 21 = 18 \\ \quad \quad \quad 126 \div 7 = 18 \end{array}$$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

Children need to be reminded that the decimal point does not move, it's the digits that move into the different place values.

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

THE FOLLOWING ARE METHODS THAT WE EXPECT THE MAJORITY OF CHILDREN TO BE ABLE TO ACHIEVE.

Stage 1

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

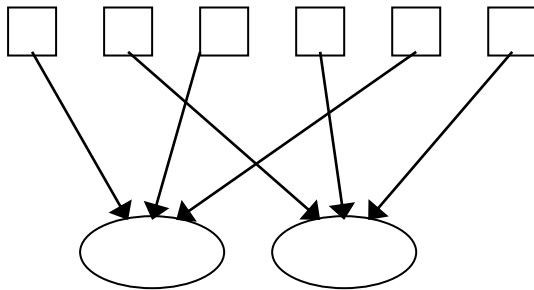


Stage 2

Children will develop their understanding of division and use jottings to support calculation

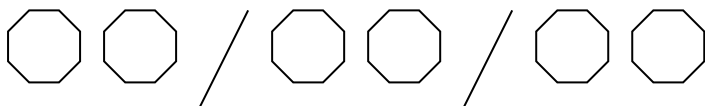
✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



✓ **Grouping or repeated subtraction**

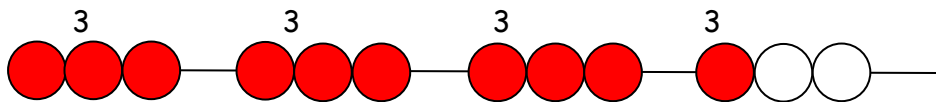
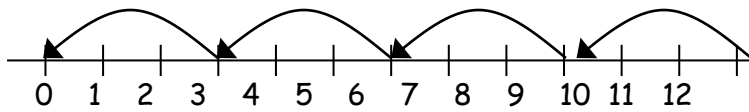
There are 6 sweets, how many people can have 2 sweets each?



Stage 3

- ✓ Repeated subtraction using a number line or bead bar

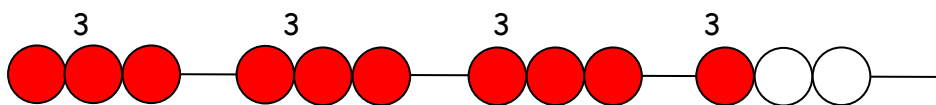
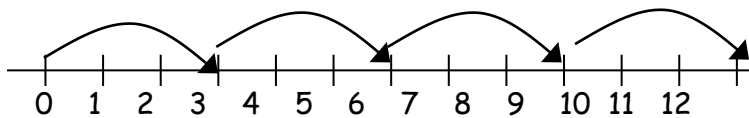
$$12 \div 3 = 4$$



Or

- ✓ Repeated addition using a number line or bead bar

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

Stage 4

Missing numbers - using the inverse

- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \div 2 = 4$$

$$20 \div \triangle = 4$$

$$100 \div \triangle = 4$$

Stage 5

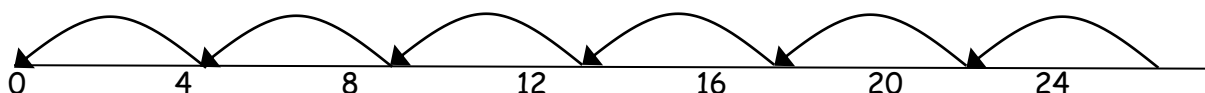
Ensure that the emphasis is on grouping rather than sharing.

Children will continue to use:

- ✓ **Repeated subtraction or addition using a number line**

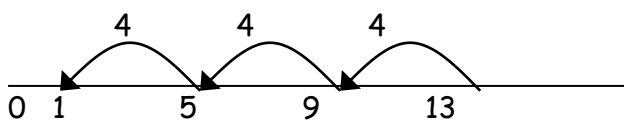
Children will use an empty number line to support their calculation.

$$24 \div 4 = 6$$



Children move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r } 1$$



Stage 6

Chunking Method

$$\begin{array}{r} 256 \div 7 = \\ 140 \quad (20 \times 7) \\ \underline{70} \quad (10 \times 7) \\ \text{Running Total} \quad 210 \\ \underline{35} \quad (5 \times 7) \\ \text{Running Total} \quad 245 \\ \underline{7} \quad (1 \times 7) \\ \text{Final Total} \quad 252 \quad 36 \end{array}$$

Chunking Table

X 7

$$20 \times 7 = 140$$

$$10 \times 7 = 70$$

$$5 \times 7 = 35$$

$$2 \times 7 = 14$$

$$1 \times 7 = 7$$

By doing a chunking table the children have already got the chunks ready to use.

Answer 36 remainder 4

Check $(36 \times 7) + 4 = 256$

Answer: 36 remainder 4 or $36 \frac{4}{7}$

Stage 7

Short division

$$284 \div 6$$

Remind children that this is the only time they will start with the highest place value.

$$\begin{array}{r}
 47 \text{ r}2 \\
 6 \overline{) 2844}
 \end{array}$$

Answer 47 r 2

Remainder as a fraction

47 r2

$$47 \frac{2}{6} \quad \text{or} \quad 47 \frac{1}{3}$$

Remainders as Decimals (the question will normal say to how may decimal places)

To One decimal place

47.3

Stage 8

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

$$87.5 \div 7$$

$$\begin{array}{r}
 12.5 \\
 7 \overline{) 87.5}
 \end{array}$$

Answer : 12.5

Stage 9

Long Division with remainders

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{30} \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

Start with the highest place value use the first 2 digits, 43 divided by 15 is 2. 2 goes in the answer, 2 x 15 is 30 that is placed under the first two digits. Then the difference between those numbers is worked out and the next number is brought down to make a new number. Then the process starts again until all numbers have been used. If there is no more numbers to be brought down this is written as a remainder.

Answer: 28 r ¹²

Long Division remainders as a fraction

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{30} \\
 132 \\
 \underline{120} \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

$$\frac{12}{15} = \frac{4}{5}$$

Using the same process:
 The remainder can be changed into a fraction.
 Simplify the fraction if possible.

Long Division remainders as a decimal

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{30} \\
 132 \\
 \underline{120} \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28.8

Using the same process, instead of leaving the remainder add a decimal point and a zero. The zero then gets brought down to carry on the process to calculate the remainder as a decimal.

+ - + - + - + - + - + - +

By the end of all stages children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

- Children should not be made to go onto the next stage if:
- 9) they are not ready i.e. lack of understanding of place value.
 - 10) they are not confident with previous stages